Deduction of Pure Spin Current from Spin Linear and Circular Photogalvanic Effect in Semiconductor Quantum Wells

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We study the spin photogalvanic effect in two-dimensional electron system with structure inversion asymmetry by means of the solution of semiconductor optical Bloch equations. It is shown that a linearly polarized light may inject a pure spin current in spin-splitting conduction bands due to Rashba spin-orbit coupling, while a circularly polarized light may inject spin-dependent photocurrent. We establish an explicit relation between the photocurrent by oblique incidence of a circularly polarized light and the pure spin current by normal incidence of a linearly polarized light such that we can deduce the amplitude of spin current from the measured spin photocurrent experimentally. This method may provide a source of spin current to study spin transport in semiconductors quantitatively.

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I. INTRODUCTION

Spin-coherent transport of conduction electrons in semiconductor heterostructures is currently an emerging subject due to its possible application in a new generation of electronic devices.[1] There have been considerable efforts to achieve spin-polarized current or pure spin current (PSC) in semiconductors. Optical injection of spin current is based largely on the fact that the spin-polarized carriers in conduction band can be injected in semiconductors via absorption of the polarized light. In the case of semiconductors, if the photon energy is higher than the characteristic energy gap, such as that of the conduction and valence bands of electrons, or of intersubband, electrons are pumped into the conduction band from the valence band or conduction subband. When the system breaks the inversion symmetry, the single-photon absorption may generate spin current or spin polarized current. The circular photogalvanic effect (CPGE), which is based on converting the helicity of light into an electric current by irradiation of circularly polarized light, was studied extensively.[2,3,4,5,6,7,8,9,10] Conventional CPGE focus only on the charge, not spin aspect of electronic transport in semiconductors. It was first realized that quantum interference of one- and two-photon excitation of unbiased semiconductors may yield ballistic spin-polarized current, which was observed by two groups.[11,12,13] Recently it was proposed that one-photon absorption of linearly polarized light should produce PSC in the noncentrosymmetric semiconductors.[14,15,16,17] On the other hand, experimental detection of PSC has been realized by measuring spin accumulation near the boundary of samples[18,19] and electric current induced by PSC in a crossbar system.[20,21,22]

Structure inversion asymmetry (SIA) in semiconductor heterojunctions may lead to spin splitting of the conduction band in the momentum **k**-space, and induce the spin-orbit coupling. This type of the systems may be one of good candidates to implement spin-based electronic devices and has attracted more and more attentions. In this paper, we investigate how to deduce PSC in the semiconductor quantum wells (QWs) by irradiation of linearly and circularly polarized lights. By using the solution of the semiconductor optical Bloch equations, we establish an explicit relation between spin photocurrent by oblique incidence of a circularly polarized light and PSC with in-plane spin polarization by normal incidence of a linearly polarized light with the same frequency and intensity. Since the photocurrent can be measured experimentally, we can deduce PSC from measured photocurrent based on several material specific parameters. This method can provide an efficient source for generating PSC quantitatively, and has potential applications in semiconductor spintronics.

II. MODEL AND GENERAL FORMALISM

We consider a QW of zinc-blende-type semiconductors with SIA. The conduction electrons can be modeled as

$$H_c = \frac{\hbar^2 k^2}{2m^*} - \lambda\hbar\left(k_x\sigma_y - k_y\sigma_x\right), \tag{1}$$

where σ are the Pauli matrices, λ is the strength of Rashba spin-orbit coupling, and m^* is the effective mass of conduction electron. The valence bands near the Γ point are described approximately by the Luttinger Hamiltonian for spin $S = 3/2$ holes,

$$H_L = -\frac{\hbar^2}{2m}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)k^2 - 2\gamma_2\left(\mathbf{k}\cdot\mathbf{S}\right)^2\right], \tag{2}$$

where γ_1, γ_2 are two Kohn-Luttinger parameters, m is the free electron mass and $\mathbf{S}$ represents three 4×4 spin

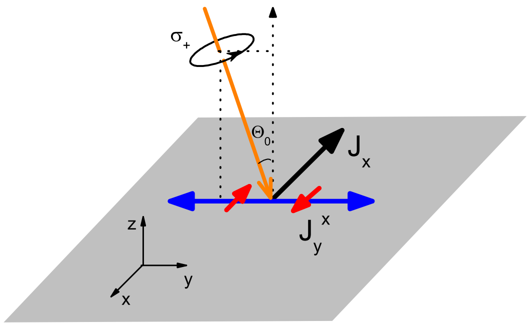

FIG. 1: The sketch of a right-handed circularly polarized (σ_+) light irradiating on the surface of a semiconductor QW in the (yz) plane with incidence angle Θ_0. In this case, the photocurrent J_x is injected perpendicularly to the incident plane of the light, and PSC J_y^x is also injected. Orange arrow denotes the direction of light propagation; thick black arrow denotes photocurrent; blue arrows denote PSC with in-plane spin polarization represented by red arrows.

3/2 matrices. For a bulk system, both heavy- and light-hole bands are degenerate at the Γ point. In a QW with thickness L, while k_x and k_y are good quantum numbers, the confinement along the z-axis is approximately realized by taking $\langle k_z\rangle = 0$, and $\langle k_z^2\rangle \simeq (\pi/L)^2$ for the lowest energy band. In the case of $k^2 = k_x^2 + k_y^2 \ll \langle k_z^2\rangle$, the energy spectrum of the first doubly degenerated heavy-hole band is reduced approximately into $E^{HH} \simeq -\hbar^2 k^2/(2m_{HH}) - \varepsilon$, with the effective mass $m_{HH} = m/(\gamma_1+\gamma_2)$, and $\varepsilon = \hbar^2\langle k_z^2\rangle(\gamma_1 - 2\gamma_2)/(2m)$. Finite thickness of the QW makes the band structure into a sequence of quasi-two-dimensional (2D) subbands with $\langle k_z^2\rangle \simeq (n\pi/L)^2$ (n is a non-zero integer), which can be calculated numerically.[23] Of course, for the precise calculations, we need to take into account the band structure of the whole $\mathbf{k}$-space. In the present paper, we first consider this simplified 2D model and then present numerical results by taking into account the finite thickness effect of band structure near the Γ point.

Now we come to study the irradiation of a polarized light on the system with incidence angle Θ_0 in the plane (yz) as shown in Figure 1. The pump pulse is of the form

$$\mathbf{E}(t) = \mathbf{E}_\omega e^{-i(\omega t - k\cos\Theta_0 z + k\sin\Theta_0 y)} + c.c., \qquad (3)$$

where ω is the frequency of the light. By treating the field perturbatively, and assuming fast interband dephasing, the semiconductor optical Bloch equations give the single particle density matrix in conduction bands due to optical irradiation[14,24,25,26]

$$\begin{aligned}\rho_{cc'}(\mathbf{k}) &= \frac{\pi e^2}{\hbar^2}\sum_v \frac{\mathbf{E}_\omega\cdot\mathbf{v}_{cv}}{\omega_{cv}}\frac{\mathbf{E}_\omega^*\cdot\mathbf{v}_{vc'}}{\omega_{c'v}} \\ &\quad\times\left[\delta(\omega-\omega_{cv}) + \delta(\omega - \omega_{c'v})\right]\tau_e, \qquad (4)\end{aligned}$$

where the subscripts c and v refer to conduction and valence bands, $\mathbf{v}_{cv}(\mathbf{k}) = \langle c\mathbf{k}|\,\mathbf{v}\,|v\mathbf{k}\rangle$ is the interband matrix element of the velocity operator, τ_e is the momentum relaxation time as a result of all various interactions, $\hbar\omega_{c(c')v} = \hbar^2k^2/(2\mu) \pm \lambda\hbar k + \Delta_0$ (with Δ_0 being the band gap and the reduced mass $\mu = m^* m_{HH}/(m^* + m_{HH})$) for the simplified 2D model. Using this solution, a physical observable $\mathbf{O}$ in conduction bands can be calculated by

$$\mathbf{O} = \sum_{c,c',\mathbf{k}} \langle c'\mathbf{k}|\,\hat{O}\,|c\mathbf{k}\rangle\,\rho_{cc'}(\mathbf{k}), \qquad (5)$$

where $\hat{O}$ is the corresponding operator. In the following, spin current operator $\hat{J}_i^j$ is defined conventionally as $\hat{J}_i^j = \frac{\hbar}{4}\{v_i, \sigma_j\}$.

III. SPIN CIRCULAR PHOTOGALVANIC EFFECT (SCPGE)

Spin photocurrent in the CPGE was studied extensively. Here we focus on spin aspect of the CPGE. Consider oblique incidence of a circularly polarized light onto the system. In this case a spin photocurrent can be circulated to be perpendicular to the incident plane of the light. When the light enters into the sample, due to the refraction effect, the light becomes $E_x = E_0 t_s\cos\varphi$, $E_y = iE_0 t_p \sin\varphi\cos\Theta$, and $E_z = iE_0 t_p\sin\varphi\sin\Theta$, where E_0 is the electric field amplitude in vacuum, Θ is the angle of refraction defined by $\sin\Theta = \sin\Theta_0/n$ (n is the index of refraction), $t_s = 2\cos\Theta_0/(\cos\Theta_0 + n\cos\Theta)$ and $t_p = 2\cos\Theta_0/(n\cos\Theta_0 + \cos\Theta)$ are transmission coefficients after Fresnel's formula for linear s and p polarizations.[27] The helicity of the incident light is $P_{\text{circ}} = (I_{\sigma_+} - I_{\sigma_-})/(I_{\sigma_+} + I_{\sigma_-}) = \sin 2\varphi$, where I_{σ_+} and I_{σ_-} are intensities of right- (σ_+) and left-handed (σ_-) polarized radiations. $P_{\text{circ}} = \pm 1$ denotes right and left circularly polarized light, respectively. In this way the photocurrent can be calculated explicitly.[26] The hole current induced in the valence bands is neglected because the effective mass of holes is typically much greater than that of electrons, and the kinetic energy and speed of holes are much less than those of the electrons.[28] In the oblique incidence of a circularly polarized light in (yz) plane, the formula (5) gives the photocurrent $J_y = 0$ and

$$J_x = -\frac{2\lambda\mu^3\Omega}{3\hbar^5\langle k_z^2\rangle m^*} t_s t_p a_0^2 e^3 E_0^2 \tau_e P_{\text{circ}}\sin\Theta, \qquad (6)$$

where $\Omega = \lambda^2\mu + \hbar\omega - \Delta_0$, and $a_0 = \sqrt{6}\langle 0,0|\,x\,|1,-1\rangle$ which is a parameter determined by experiment. It is clear that the photocurrent circulates only in the case of the circularly polarized light ($P_{\text{circ}} \neq 0$), and vanishes in the case of linearly polarized light ($P_{\text{circ}} = 0$). Besides the photocurrent in CPEG, a PSC with x-component spin polarization perpendicular to the direction of photocurrent also circulates,

$$J_y^x = (I_+^C t_p^2\cos^2\Theta\sin^2\varphi + I_-^C t_s^2\cos^2\varphi)\hbar a_0^2 e^2 E_0^2\tau_e, \qquad (7)$$

where

$$I_{\pm}^{C} = \frac{\lambda\mu}{6\hbar^5 \langle k_z^2\rangle m^*}\left[\hbar^2 \langle k_z^2\rangle (m^* - \mu) \pm \mu^2\Omega\right]. \quad (8)$$

The spin current even survives even in the normal incidence while the photocurrent vanishes. It is sketched in Figure 1 that the photocurrent J_x and PSC J_y^x are induced by a right-handed circularly polarized light irradiating on the surface of a semiconductor QW in the (yz) plane with incidence angle Θ_0.

Absorption of a circularly polarized light in semiconductors induces z-component spin polarization S^z due to the conservation of angular momentum. The light-induced non-zero S^z will lead to an orientational distribution of PSC with the z-component polarization

$$J_r^z(\theta) = -\frac{\mu^2\Omega}{6\hbar^3 m^*\pi\delta} t_s t_p \hbar a_0^2 e^2 E_0^2 \tau_e P_{\mathrm{circ}} \cos\Theta, \quad (9)$$

with $\theta = \arctan(k_x/k_y)$, $\delta = \sqrt{\mu(\lambda^2\mu + 2\hbar\omega - 2\Delta_0)}$, and the subscript r denoting the radial direction in polar coordinates. However, one has $J_{x(y)}^z(\mathbf{k}) = J_{x(y)}^z(-\mathbf{k})$ such that the total spin current with z-component polarization vanishes. With the geometric constraint of the sample, a PSC of z-component polarization can circulate and may be used to implement the reciprocal spin Hall effect.[29] In the case of normally incident σ^+ polarized light, the orientational distributions of radial spin current with tangent direction polarization (i.e., $J_r^\theta(\theta)$) and tangent spin current with radial direction polarization (i.e., $J_\theta^r(\theta)$) are given by

$$J_r^\theta(\theta) = \frac{\lambda\mu(2\mu - m^*)}{12\hbar^3 m^*\pi} t_0^2 \hbar a_0^2 e^2 E_0^2 \tau_e, \quad (10)$$

$$J_\theta^r(\theta) = \frac{\lambda\mu}{12\hbar^3\pi} t_0^2 \hbar a_0^2 e^2 E_0^2 \tau_e, \quad (11)$$

where the sub- and super-script θ denotes the tangent direction in polar coordinates. The total contribution of the orientational distributions of spin current leads to a non-vanishing spin current with in-plane spin polarization as shown in Eq. (7).

For the sake of clarity, the orientational distributions of spin current in the case of normally incident σ^+ polarized light are plotted in Figure 2.

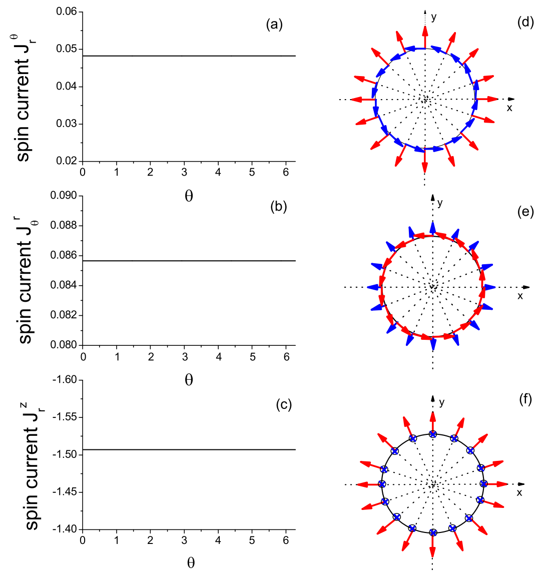

FIG. 2: Orientational distribution of pure spin current in the case of normally incident σ_+ polarized light. (a-c) for numerical calculation results (in unit of 10^{-3}eV^{-2}.nm^{-1}.fs^{-2}.$\hbar a_0^2 e^2 E_0^2 \tau_e$). The experiment parameters are taken as: $\Delta_0 = 750$meV, $\lambda = 6.3$meV.nm/$\hbar$, $m^* = 0.05m$, $\gamma_1 = 6.9$, $\gamma_2 = 2.2$, $L = 14$nm, the index of refraction $n = \sqrt{13}$, and the wave length of the light $\lambda_0 = 1620$nm. (d-f) for the sketches of orientational distribution, in which red arrows denote the directions of spin current, blue arrows denote the polarization directions of spin current, and $\otimes$ denotes spin polarization along $-z$ direction. (a) and (d) for $J_r^\theta(\theta)$; (b) and (e) for $J_\theta^r(\theta)$; (c) and (f) for $J_r^z(\theta)$.

IV. SPIN LINEAR PHOTOGALVANIC EFFECT (SLPGE)

Now we come to consider the normal incidence of a linearly polarized light onto the sample, and the pump pulse is of the form

$$\mathbf{E}(t) = \mathbf{E}_\omega \exp[-i(\omega t - kz)] + c.c. \quad (12)$$

In the medium, $E_x = t_0 E_0 \cos\phi$ and $E_y = t_0 E_0 \sin\phi$, with $t_0 = 2/(1+n)$ and ϕ is the angle between the polarization plane and the x-axis, e.g. $\phi = 0$ corresponding to the x polarized light. In this case it was known that no photocurrent is injected as in Eq. (6). However, a PSC may survive. The physical origin of spin current is given briefly as follows: Due to Rashba spin-orbit coupling, conduction band splits into two subbands denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. When the frequency ω of the light satisfies the condition $\hbar\omega > \Delta_0$, electrons are pumped from the heavy-hole band into conduction bands. If there appears a electron state $|\mathbf{k},\uparrow\rangle$ in conduction band $|\uparrow\rangle$ with momentum $\mathbf{k}$, $|-\mathbf{k},\downarrow\rangle$ must appear in conduction band $|\downarrow\rangle$ with momentum $-\mathbf{k}$ with the same probability according to the symmetry. $|\mathbf{k},\uparrow\rangle$ and $|-\mathbf{k},\downarrow\rangle$ are two degenerate states and have opposite velocities, thus the pair contributes a null electric current. However, they have opposite spin polarization such that they carry equal spin current. Therefore a finite spin current survives for these two states. The spin splitting of conduction band plays an essential role in this mechanism.[21,22] As an example, the orientational distributions of spin current in the case of normally incident x polarized light are plotted in Figure 3.

An explicit calculation gives PSC with in-plane spin polarization,

$$J_x^y = \left(-I_0^L - I_1^L \cos 2\phi\right) t_0^2 \hbar a_0^2 e^2 E_0^2 \tau_e, \quad (13)$$

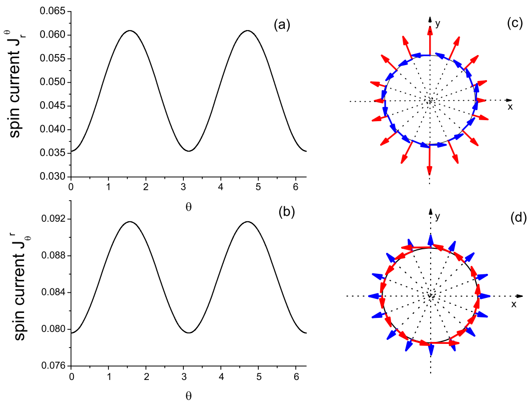

FIG. 3: Orientational distribution of pure spin current in the case of normally incident x polarized light. (a) and (b) for numerical calculation results (in unit of 10^{-3}eV^{-2}.nm^{-1}.fs^{-2}.$\hbar a_0^2 e^2 E_0^2 \tau_e$). The experimental parameters are taken as the same as those given in Fig. 2. (c) and (d) for the sketches of orientational distribution, in which red arrows denote the directions of spin current, and blue arrows denote the polarization directions of spin current. (a) and (c) for $J_r^\theta(\theta)$; (b) and (d) for $J_\theta^r(\theta)$.

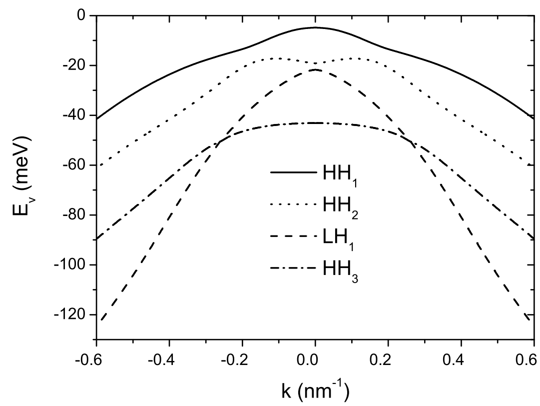

FIG. 4: Dispersion of low lying subbands for the Luttinger effective Hamiltonian with the width $L = 14$nm, $\gamma_1 = 6.9$ and $\gamma_2 = 2.2$. HH and LH denote heavy hole and light hole, respectively.

$$J_x^x = I_1^L t_0^2 \hbar a_0^2 e^2 E_0^2 \tau_e \sin 2\phi, \tag{14}$$

where $I_0^L = \lambda\mu\left(m^* - \mu\right)/\left(6\hbar^3 m^*\right)$ and $I_1^L = \lambda\mu^3\Omega/\left(6\hbar^5 m^* \left\langle k_z^2\right\rangle\right)$. It is obvious that PSC with an in-plane spin polarization is dependent on the angle ϕ between the polarization plane and the x-axis. A linearly polarized light can be decomposed as a combination of two circularly polarized beams of light. The phase difference between these two composite beams of the light is 2ϕ. The polarization dependence of the PSC originates from the interference of two composite circularly polarized lights.

V. RELATION BETWEEN PHOTOCURRENT AND SPIN CURRENT

The two formulas for photocurrent in Eq. (6) and spin current in Eq. (13) contain the parameter a_0 and the relaxation time τ_e which need to be determined experimentally. Assume the intensity and the frequency of the two applied lights are equal, the ratio of the photocurrent of circularly polarized light with an oblique angle Θ_0 to the spin current of normally incident linear polarized light gives

$$\frac{J_x}{J_x^y} = \eta \frac{t_s t_p}{t_0^2} P_{\mathrm{circ}} \sin\Theta \frac{2e}{\hbar}, \tag{15}$$

where η is a dimensionless frequency-dependent factor,

$$\eta = \frac{2\Omega}{\epsilon_0 + \Omega\cos 2\phi}, \tag{16}$$

with $\epsilon_0 = \hbar^2\left\langle k_z^2\right\rangle\left(m^* - \mu\right)/\mu^2$. For a small incidence angle Θ_0 and $P_{\mathrm{circ}} = 1$, the ratio is reduced to $J_x/J_x^y \approx \eta\frac{\Theta_0}{n}\frac{2e}{\hbar}$. In this way we establish an explicit relation between light-injected photocurrent and PSC. All parameters in η are known in semiconductor materials. For the sample of InGaAs,[10] for instance, $\Delta_0 = 750$meV, $\lambda = 6.3$meV.nm/$\hbar$, $m^* = 0.05m$, $\gamma_1 = 6.9$, $\gamma_2 = 2.2$, $L = 14$nm, $\epsilon_0 = 50.8$meV, and $\Omega = \hbar\omega - 749.982$meV. In this system, the photocurrent J_x in CPGE has been measured successfully. Therefore we can deduce the spin current by measuring the photocurrent experimentally.

VI. NUMERICAL RESULTS

The formula in Eq. (16) is only valid for excitation of electrons near the Γ point. In principle we can calculate the ratio of photocurrent to spin current following the $\mathbf{k}\cdot\mathbf{p}$ calculation done by Baht et al.[14] Here we present our results after the quantum size effect of QW with a finite thickness L is taken into account. For a confining potential $V(z)$ along the z-axis, say $V(z) = +\infty$ for $|z| > L/2$ and $V(z) = 0$ for otherwise. While k_x and k_y remain to be good quantum numbers, the quantization along the z-axis can be calculated numerically by the truncation approximation if L is of order of tens nm.[22,30] The lowest four valence subbands are plotted in Figure 4, where each subband is doubly degenerated.

In this way photocurrent and PSC can be calculated numerically in terms of the unknown parameters a_0 and τ_e. The variations of J_x in the case of oblique incident σ_+ polarized light with the frequency of light is plotted in Figure 5(a), and the photocurrent has its sign change when the dominant contribution of interband transition to the conduction band switches from the first heavy-hole sub-band to the second heavy-hole sub-band. We also plot the spin current J_x^y in a normally incident x polarized light in Figure 5(b). The frequency dependence of the dimensionless factor η is plotted in Figure 5(c). When $\hbar\omega$ is close to the band gap Δ_0 the main contri-

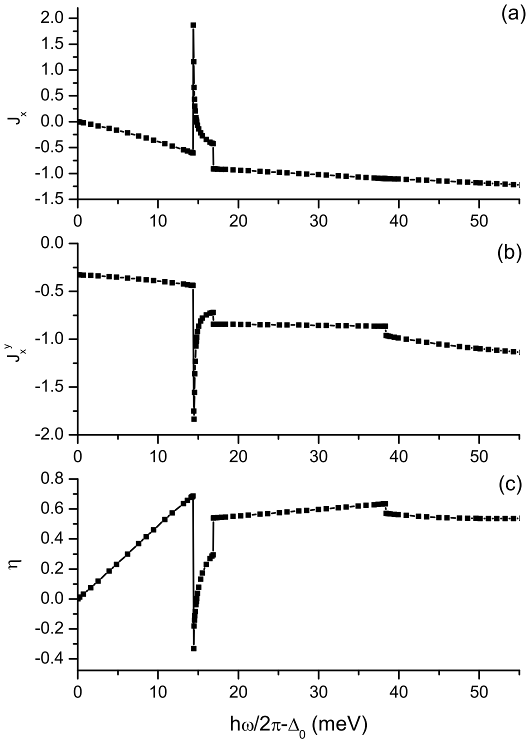

FIG. 5: Numerical results based on the model of QW with a finite thickness L. The parameters are given in the text following Eq. (12). (a) Spectrum of photocurrent J_x in the case of oblique incident σ_+ polarized light (in unit of 10^{-3}eV^{-2}.nm^{-1}.fs^{-2}.$t_s t_p a_0^2 e^3 E_0^2 \tau_e \sin\Theta$); (b) Spectrum of spin current J_x^y in a normally incident x polarized light (in unit of 10^{-3}eV^{-2}.nm^{-1}.fs^{-2}.$t_0^2 \hbar a_0^2 e^2 E_0^2 \tau_e$); (c) The frequency dependence of the dimensionless factor $\eta(\omega)$.

bution results from only interband transition from the first heavy-hole subband to the conduction band, the ratio factor is linear in the frequency ω. The photocurrent was observed experimentally in the two samples of InGaAs with Rashba coupling $\lambda_1 = 3.0$meV.nm/$\hbar$ and $\lambda_2 = 6.3$meV.nm/$\hbar$.[10] The photocurrent changes its sign when the frequency of laser increases. The angle dependence of photocurrent gives $J_x(\Theta_0) \simeq 351\Theta_0/n$ nA for a small angle Θ_0 (with the index of refraction $n = \sqrt{13}$). The ratio factor is estimated as $\eta \simeq 0.62$. If we keep our conditions of laser except that the helicity of light changes from circular to linear, the PSC in the linearly polarized light is estimated as $J_x^y \simeq 566\hbar/2e$ nA.

VII. CONCLUSION

Here we use the model with the twofold conduction band described by Rashba coupling and the valence band by the Luttinger Hamiltonian to investigate spin photogalvanic effect induced by polarized lights via interband excitations in the semiconductor QWs. We established a relation between light-injected photocurrent and PSC. As the photocurrent in CPGE was extensively studied both theoretically and experimentally, we can make use of it to deduce PSC in the same system by using the linearly polarized light to replace the circularly polarized light in CPGE, which can be realized by adding a 1/4-wave plate. Thus this method may provide a source of spin current to study spin transport in semiconductors quantitatively.

Acknowledgments

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